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TRANSIENT HEAT TRANSFER FROM A WIRE IN A VIOLENTLY FLUCTUATING ENVIRONMENT

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Abstract—A closed form perturbation solution is obtained for the problem of transient heat transfer from a constant temperature circular cylinder in a violently fluctuating flow of incompressible Newtonian fluids. The small parameter used in the solution is the ratio of the maximum fluctuation amplitude to the cylinder diameter. The solution is asymptotically valid for any values of the Prandtl and Reynolds numbers. The natural convection is considered as the second-order effect. The solution is also applied to predict the initial transient heat transfer from a wire which starts from rest an arbitrary translational motion. Numerical results in terms of time-dependent local and average Nusselt numbers are used to demonstrate the significant time lag of heat transfer response to the fluctuating velocity. It is shown that an optimum frequency of sinusoidal oscillation exists for the maximum net heat transfer at given flow parameters.

NOMENCLATURE

a	radius of cylinder
C_p	fluid specific heat
g_1	dimensionless gravitational acceleration, $g\bar{t}/(\nu\delta/a^2)$
g	gravitational acceleration
$\mathbf{i}$	unit vector in x-direction
$\mathbf{j}$	unit vector in y-direction
k	fluid thermal conductivity
P	pressure
q	wall heat flux per unit area
r	polar radial distance
T	temperature
T_a	surface temperature of cylinder
T_∞	ambient temperature
ΔT	temperature difference, $T_a - T_\infty$
t	time
$\bar{t}$	characteristic time
U	cylinder velocity
$\dot{U}$	cylinder acceleration
U_0	mean flow velocity
U_m	maximum cylinder velocity
U_∞	free stream velocity
u	dimensionless cylinder velocity
$\mathbf{V}$	fluid velocity
$\mathbf{v}$	dimensionless fluid velocity
x, y	Cartesian coordinates (Fig. 1)

Greek symbols

α	dimensionless thermal diffusion time, τ/Pr
β	coefficient of thermal expansion
γ	Euler's constant, 0.5772157
Θ	dimensionless temperature
θ	polar angle
ε	dimensionless fluctuation amplitude, δ/a
μ	fluid dynamic viscosity
ν	fluid kinematic viscosity
ρ	fluid density
τ	dimensionless time, $\nu t/a^2$
τ_c	time interval of uniform acceleration
Φ	dissipation function
ϕ	dimensionless dissipation function, $\Phi/(\delta^2/a^2\bar{t}^2)$
χ	fluid thermal diffusivity
ω	dimensionless frequency, $(a^2/\nu)\omega_1$
ω_1	angular frequency
∇	dimensionless gradient operator, $a\nabla_1$
∇_1	dimensional gradient operator

Dimensionless group

Ec	thermal Eckert number, $\nu/C_p\Delta T\bar{t}$
Gr	thermal Grashof number, $g\beta D^3\Delta T/\nu^2$
$\mathbf{Gr}$	Grashof vector, $[\beta a\Delta T(\mathbf{i}\dot{U} - \mathbf{j}g)]/(\delta/\bar{t})^2$
Nu	local Nusselt number, $q/(k\Delta T/a)$
$\overline{Nu}$	average Nusselt number, $\int_1^{2\pi} \nu \, d\theta$
Pe	Peclet number, $Pr\, Re_N$
Pr	fluid Prandtl number, $\nu/\chi = \mu C_p/k$
Re	Reynolds number, $a^2/\nu\bar{t}$
Re_0	mean flow Reynolds number, $2U_0 a/\nu$
Re_N	fluctuation Reynolds number, $\varepsilon\, Re$

Subscripts

0	zeroth-order solution
1	first-order solution
2	second-order solution
m	maximum value

1. INTRODUCTION

A GREAT deal has been learned about the transient heat transfer from a heated circular cylinder which impulsively starts from rest a constant velocity translation [1–5]. These theoretical results cannot be tested precisely with experiments, because it is very difficult, if not impossible, to produce in laboratories an impulsively started motion. On the other hand a great deal of experimental studies [6–16] have been reported on the effects of sinusoidal oscillation on the heat transfer from a cylinder. These results remain largely unaccompanied by theoretical studies, due to the analytical and numerical difficulties associated with the large amplitude oscillation. Only a few theoretical studies on the effect of small amplitude sinusoidal oscillation are known [5, 17].

The purpose of our work is to fill in this apparent information gap. As a start, we give a method of predicting transient heat transfer from a circular cylinder which starts from rest a small amplitude fluctuation of an arbitrary form. The same results are also applied to predict the initial transient heat transfer for the cases of realistically producible cylinder motions. Some numerical results based on the present theory are compared with the known results. New results are also given. A similar theory without any consideration of the natural convection effect was given earlier by Lin [18]. However, he did not give any numerical results.

2. MATHEMATICAL SOLUTION

Consider the heat transfer from a constant temperature circular cylinder of radius a fluctuating with an instantaneous velocity $-\mathbf{i}U(t)$ in an initially quiesient fluid as shown in Fig. 1. The fluid is Newtonian and incompressible. Let the characteristic time and amplitude of the cylinder fluctuation be respectively $\bar{t}$ and δ. Let the dimensionless velocity, pressure, gradient operator, temperature and time be respectively related to their dimensional counter parts $\mathbf{V}$, P, ∇_1, T and t by

$$v = V(\bar{t}/\delta), \quad p = P(\bar{t}/\delta)^2/\rho_\infty, \quad \nabla = a\nabla_1,$$

$$\Theta = (T-T_\infty)/(T_a-T_\infty) = (T-T_\infty)/\Delta T, \quad \tau = t(v/a^2),$$

where ρ_∞ is the ambient density, T_∞ is the ambient temperature, T_a is the surface temperature of the cylinder and v is the kinematic viscosity. The dimensionless governing equations of mass, momentum and energy with respect to a reference frame

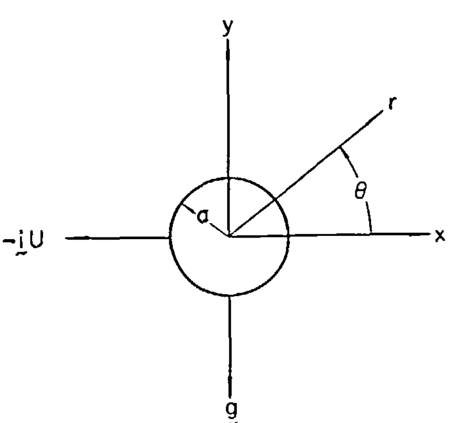

FIG. 1. Geometry and coordinate system.

attached to the cylinder are

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\partial \mathbf{v}/\partial \tau + \varepsilon\, Re(\mathbf{v}\cdot\nabla)\mathbf{v} = -\varepsilon\, Re\, \nabla p + \nabla^2 \mathbf{v} - \varepsilon\, Re\, \mathbf{Gr}\, \Theta, \tag{2}$$

$$Pr\, \partial\Theta/\partial\tau + \varepsilon\, Re\, Pr\, \mathbf{v}\cdot\nabla\Theta = \nabla^2\Theta + \varepsilon^2\, Re\, Pr\, Ec\, \phi, \tag{3}$$

where $\varepsilon = \delta/a$, $\phi = \Phi/(\delta/a\bar{t})^2$ Φ being the dissipation function, and Pr, Re, and $\mathbf{Gr}$ the Prandtl number, the Reynolds number and the Grashof vector, respectively.

Equation (1) is automatically satisfied by the stream function ψ which is related to the r and θ velocity components in the cylindrical coordinates by

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta}, \quad v_\theta = \frac{\partial\psi}{\partial r}.$$

In terms of ψ, equations (2) and (3) can be written as

$$\left(\frac{\partial}{\partial\tau} - \nabla^2\right)\nabla^2\psi = \varepsilon G_1\left[(\dot{u}\sin\theta + g_1\cos\theta)\frac{\partial\Theta}{\partial r} + \frac{1}{r}(\dot{u}\cos\theta - g_1\sin\theta)\frac{\partial\Theta}{\partial\theta}\right] - \varepsilon\frac{Re}{r}\frac{\partial(\psi,\nabla^2\psi)}{\partial(r,\theta)}, \tag{4}$$

$$\left(Pr\frac{\partial}{\partial\tau} - \nabla^2\right)\Theta = -\varepsilon\frac{Re\,Pr}{r}\frac{\partial(\psi,\Theta)}{\partial(r,\theta)}, \tag{5}$$

where the upper dot denotes differentiation with τ, and

$$u(\tau) = U(t)/(\delta/\bar{t}), \quad G_1 = \beta\Delta T/\varepsilon,$$

$$g_1 = g/(a^2\bar{t}/v\delta),$$

g being the gravitational acceleration, and

$$\frac{\partial(\psi,F)}{\partial(r,\theta)} = \frac{\partial\psi}{\partial r}\frac{\partial F}{\partial\theta} - \frac{\partial\psi}{\partial\theta}\frac{\partial F}{\partial r},$$

is the Jacobian with F standing for $\nabla^2\psi$ or Θ. The initial conditions for equations (4) and (5) are

$$\psi(r,\theta,\tau) = 0 \quad \text{and} \quad \Theta(r,\theta,\tau) = 0, \quad \tau \leqslant 0.$$

The boundary conditions are

$$\frac{\partial\psi}{\partial r} = 0, \quad -\frac{1}{r}\frac{\partial\psi}{\partial\theta} = 0, \quad \Theta(r,\theta,\tau) = 1 \quad \text{at } r = 1,$$

$$\frac{\partial\psi}{\partial r} = -u(\tau)\sin\theta, \quad -\frac{1}{r}\frac{\partial\psi}{\partial\theta} = u(\tau)\cos\theta \quad \text{at } r \to \infty.$$

The formulated initial-boundary value problem will be solved with a regular perturbation series

$$\psi = \sum_{n=0} \varepsilon^n\psi_n, \quad \Theta = \sum_{n=0} \varepsilon^n\Theta_n.$$

This series solution is expected to be valid for all time for small amplitude but otherwise arbitrary fluctuation such that $\varepsilon = \delta/a \ll 1$ but Re, Pr, and $\mathbf{Gr}$ may be finite [19–21]. The same series expansion is also valid for any arbitrary $u(\tau)$ during the initial time when ε remains much smaller than one [19–21]. For the case of fluctuating flow about a cylinder, the characteristic time is $1/\omega_1$, and $Re = \omega_1 a^2/v$, where ω_1 is the characteristic frequency. The environment is said to be violently fluctuating with respect to the cylinder, if ω_1 is so large that $Re \gg 1$. We consider in this study only the cases in which

$$\mathrm{Gr} = O(1), \quad Ec = O(\varepsilon), \quad \phi = O(1) \quad \text{and } \varepsilon \ll 1.$$

For these cases, ψ_n and Θ_n are decoupled in equations (4) and (5) up to $O(\varepsilon^3)$ for any finite Re and Pr. The case of $\mathrm{Gr} = O(\varepsilon)$ was considered by Lin [18].

The zeroth-order solution for the stream function ψ_0 has already been obtained in other applications [19, 20] with a novel integral transformation of the dependent variable followed by the method of Laplace transform, and is given by

$$\psi_0 = f_0(r,\tau)\sin\theta,$$

$$f_0 = u(\tau)\left(\frac{1}{r}-r\right)+\frac{1}{r}\int_1^r x_0(s,\tau)s\,\mathrm{d}s,$$

$$x_0(r,\tau) = 2\int_0^\tau \dot{u}(\lambda)\bar{x}_0(r,\tau-\lambda)\,\mathrm{d}\lambda, \tag{6}$$

$$\bar{x}_0(r,\tau) = 1+\frac{2}{\pi}\int_0^\infty \exp(-\omega^2 r)C_0(\omega,r)\frac{\mathrm{d}\omega}{\omega},$$

$$C_0(\omega,r) = [J_0(\omega r)Y_0(\omega)-Y_0(\omega r)J_0(\omega)] \div [J_0^2(\omega)+Y_0^2(\omega)],$$

where J_0 and Y_0 are respectively the Bessel functions of the first and second kind of the zeroth-order. The zeroth-order solution for the temperature field Θ_0 can be obtained from the unsteady heat conduction equation by use of the Laplace transform, and is given by

$$\Theta_0 = 1+\frac{2}{\pi}\int_0^\infty \exp(-\omega^2\tau/Pr)C_0(\omega,r)\frac{\mathrm{d}\omega}{\omega}.$$

Higher order solutions ψ_1, ψ_2 and Θ_1, Θ_2 can be obtained respectively from the unsteady Stokes equation and heat equation with convective terms as non-homogeneous sources. The boundary conditions are all homogeneous. The reason the convective terms can be treated as known sources even at finite Re and Gr is because they are dominated by the local time terms [19–21]. The method of solution is novel but rather involved. It is documented elsewhere [22]. Only the final solutions are quoted below.

$$\psi_1 = Re\,f_{11}(r,\tau)\sin 2\theta+f'_{11}(r,\tau)\sin\theta+f'_{12}(r,\tau)\cos\theta, \tag{7}$$

$$\begin{aligned}\psi_2 = {} & Re^2[f_{2a}(r,\tau)\sin\theta+f_{2b}(r,\tau)\sin 3\theta] \\ & +Re[f_{21}(r,\tau)\sin 2\theta+f_{22}(r,\tau)\cos 2\theta \\ & +f_{23}(r,\tau)+f'_{21}(r,\tau)\sin\theta+f'_{22}(r,\tau)\cos\theta],\end{aligned} \tag{8}$$

$$\Theta_1 = Re\,Pr\,J(r,\tau)\cos\theta, \tag{9}$$

$$\begin{aligned}\Theta_2 = {} & (Re\,Pr)[X_1(r,\tau)\cos\theta-X_2(r,\tau)\sin\theta] \\ & +(Re\,Pr)^2[M(r,\tau)\cos^2\theta+N(r,\tau)\sin^2\theta \\ & +Q(r,\tau)\cos 2\theta],\end{aligned} \tag{10}$$

where all functions of r and τ are given in the Appendix. These functions will be found useful by the reader who wishes to obtain numerical results for the particular $u(\tau)$ encountered in his practice but not included in the present numerical examples.

The local heat transfer from the cylinder is

$$q = -\frac{k\Delta T}{a}\left[\frac{\partial}{\partial r}(\Theta_0+\varepsilon\Theta_1+\varepsilon^2\Theta_2)+O(\varepsilon^3)\right]_{r=1},$$

and the local Nusselt number is

$$Nu = q/(k\Delta T/a),$$

where k is the thermal conductivity. The net heat transfer from the cylinder is the integral of q from $\theta = 0$ to 2π, and the average Nusselt number is

$$\overline{Nu} = \int_0^{2\pi} Nu\,\mathrm{d}\theta.$$

It follows from equations (6)–(10) that

$$\overline{Nu} = -2\pi\Theta_{0_r}-\pi(\varepsilon\,Re\,Pr)^2[M_r+N_r]+O(\varepsilon^3), \tag{11}$$

$$\begin{aligned}-Nu = {} & \Theta_{0_r}+\varepsilon\,Re\,Pr\,J_r\cos\theta+(\varepsilon\,Re\,Pr)^2 \\ & \times[M_r\cos^2\theta+N_r\sin^2\theta+Q_r\cos 2\theta] \\ & +\varepsilon^2\,Re\,Pr[X_r\cos\theta-Y_r\sin\theta]+O(\varepsilon^3),\end{aligned} \tag{12}$$

where subscript r stands for partial differentiation with respect to r, and all functions of r are evaluated at $r = 1$.

The closed form solutions we have obtained involve time in the integrands of multiple integrals. Thus the instantaneous heat transfer depends on the entire past history of the cylinder motion and heat transfer. As a consequence, the determinations of the instantaneous Nusselt number requires the evaluation of the same multiple integrals for each τ starting from $\tau = 0$. This makes the necessary numerical computation very time consuming. This is the major disadvantage of the present method. The advantage of the method is that the same solution applies to any form of $u(\tau)$ subjected to known constraints. On the other hand any direct numerical method requires a new program for each form of $u(\tau)$. A direct numerical program for a particular form of $u(\tau)$ included in our computation does not seem to exist. It should be pointed out that equation (11) does not state that $\overline{Nu} \sim Pr^2$, since Pr also appears in the integrands of M_r and N_r. However, Re does not appear in the integrands, but appears only in the coefficient of equation (11). Thus

$$\overline{Nu}+2\pi\Theta_{0_r}(1,\tau) \sim (\varepsilon\,Re)^2$$

for any given Pr and τ. Hence $\overline{Nu}$ for any Re can be inferred from the value of $\overline{Nu}$ at any other value of Re at the same τ and Pr.

3. RESULTS AND DISCUSSIONS

Due to the limited computer time available, we obtained only the results for short time. Short-time expansions of $\bar{x}_0$ and V_1 for $\tau \ll 1$ are used in all numerical integrations by the Gauss quadrature [23].

First consider the case of a cylinder with $u(\tau)$ and $\Theta(1,\theta,\tau)$ both given by the unit step function; i.e. the motion and heating of the cylinder are both impulsively started. It follows from the nondimensionalization scheme that $\bar{t} = \nu\delta/V = \delta/U_m$ and $\varepsilon\, Re = U_m a/\nu = Re_N$ where U_m is the uniform cylinder velocity. The $\overline{Nu}$ values for the case of $Re_N = 100$ and $Pr = 0.73$ are given in Fig. 2 together with the corresponding results of Jain and Goel [1] and Sano [2]. The agreement with Sano's results obtained from the matched asymptotic expansion is better than with Jain and Goel's results obtained with the boundary layer theory.

An impulsively started motion is extremely difficult, if not impossible, to produce. In actual situations, the cylinder motion is more like that of a uniform acceleration until a finite time τ_c and then followed by a constant velocity. $\overline{Nu}$ values corresponding to two values of τ_c are given in Fig. 3 together with the results of the impulsively started case. As expected, the average Nusselt numbers of the impulsively started case bound from above those of the other two more realistic cases.

Next consider the case of sinusoidal fluctuation starting from rest. In this case

$$\bar{t} = \nu\delta/V = \sin(\omega\tau)\delta/\omega_1\delta\,\sin(\omega_1 t) = 1/\omega_1,$$

where ω_1 is the dimensional frequency. Hence $\varepsilon Re = \omega_1 \delta a/\nu = Re_N$. The results of $\overline{Nu}$ for $\omega = \omega_1(a^2/\nu) = 1300$ are given in Fig. 3. It is seen that the cylinder fluctuation at this particular value of ω cannot enhance the heat transfer beyond that corresponding to the impulsively started cylinder at the same Re_N, at least during the initial time. To study the effects of the

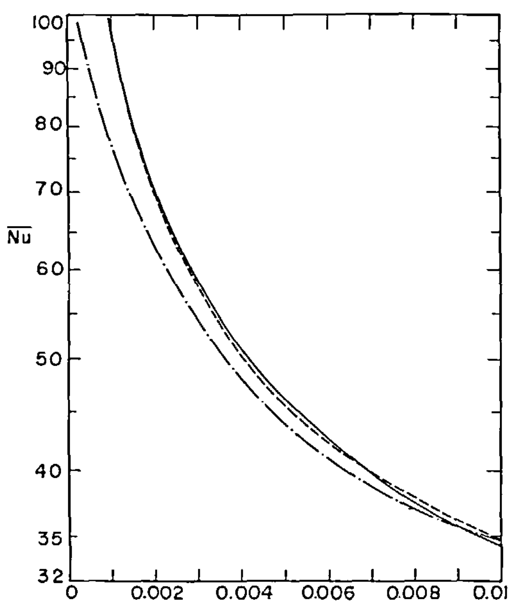

FIG. 2. Variation of average Nusselt number with time at $Pr = 0.73$ and $Re_N = 100$ for the case of an impulsively started uniform motion. ——, present study; —·—, Jain and Goel [1]; -----, Sano [2].

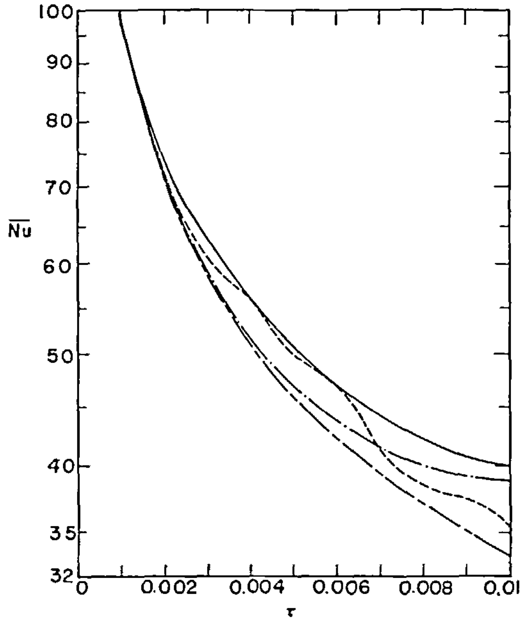

FIG. 3. Variation of average Nusselt number with time at $Re_N = 200$ and $Pr = 0.73$. ——, impulsively started uniform motion; -----, sinusoidal fluctuation at $\omega = 1300$; —·—, uniform acceleration until $\tau_c = 0.01$ then followed by a constant velocity; — — —, uniform acceleration until $\tau_c = 0.1$ then followed by a constant velocity.

fluctuation frequency on the heat transfer, we plot the fluctuating part of the Nusselt number, i.e. $\overline{Nu}_t = \overline{Nu} - \overline{Nu}_c$ in Figs. 4 and 5, where $\overline{Nu}_c = -2\pi\Theta_{0_r}(1,\tau)$ is due to transient heat conduction. $\overline{Nu}_c$ for two values of Pr are plotted in Fig. 6. As the frequency increases from 40 to 1300, the magnitude of the fluctuating heat transfer decreases dramatically. At relatively high frequencies, the oscillation in velocity appears to occur so rapidly that there is not enough time to develop a large temperature gradient close to the cylinder. Consequently, only a small erratically fluctuating quantity of heat is transferred to the ambient fluid during the transience. On the other hand, as ω decreases from 40 to 1, the fluctuating heat transfer is again sharply suppressed and approaches zero as $\omega \to 0$. Thus there must exist an optimum frequency, ω_{opt}, between $\omega = 1$ and 1300 such that $\overline{Nu}_t$ is the maximum. The heat transfer responses at $\omega = 40$, 100 and 1300 in each first cycle of oscillation lag behind the corresponding sinusoidal velocity by 0.4227π, 0.4586π, and 1.109π, respectively. The period of the first cycle is the time required for the heat transfer to reach the second minimum after the start of the cylinder oscillation. The phase lag diminishes as frequency decreases as expected. It is interesting to note in Fig. 4 that the values of $\overline{Nu}_t$ corresponding to the negative velocity are higher than those corresponding to the positive velocity during the oscillation. This pumping effect disappears in the case of higher frequency shown in Fig. 5.

Similar computations show that both the phase lag and the amplitude of the fluctuating heat transfer

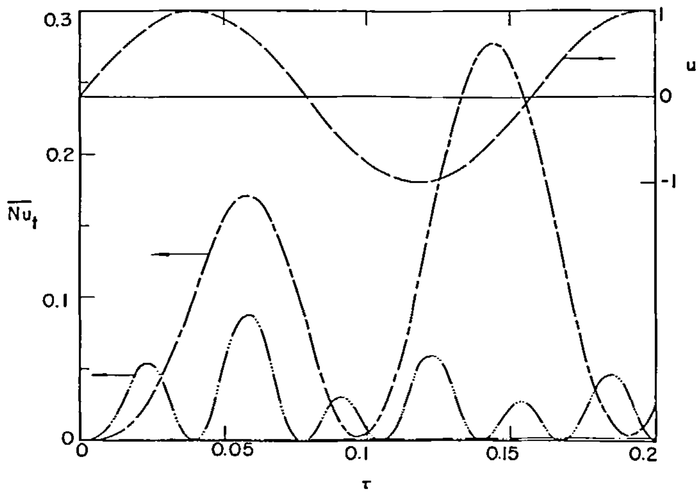

FIG. 4. Variation of fluctuating heat transfer ($\overline{Nu}_t$) with time at $Re_N = 10$ and $Pr = 0.73$. —··—, $\omega = 1$; – – – –, $\omega = 40$; —···—, $\omega = 100$. ——, $\sin(40\tau)$.

increase as Pr is increased, when ω and Re_N are held fixed [22]. This is quite to be expected, since an increase in Pr means an increase in thermal diffusion time relative to the momentum diffusion time. However, the phase lag does not vary with a change in $Re_N = \varepsilon Re$, since Re_N is not coupled with time in the expression of $\overline{Nu}_t$. As is already pointed out at the end of the last section, $\overline{Nu}_t \sim Re_N^2$.

There do not seem to exist measurements of initial transient heat transfer that we can use for direct comparisons. However, there are some known works which are related with the present study. Antonini *et al.* [16] measured the stationary heat transfer from a constant-temperature hot wire which oscillates sinusoidally in an initially quiescent fluid until the flow becomes stationary. They found that there was always a phase lag between the heat transfer fluctuation and the velocity fluctuation at finite frequencies. The same is found during the transience studied here. It appears that the phase lag which occurs during the transience diminishes but will not vanish as $\tau \to \infty$. Davies [17] found with an Oseen-type approximation that the phase lag always exists unless $\omega_1 < 0.2\pi U_0^2/\chi$, where χ is the thermal diffusivity and U_0 is the constant mean velocity of the ambient fluid in a stationary motion with a small amplitude sinusoidal oscillation. This statement implies that there will always be phase lags as $U_0 \to 0$ (which is the case in our study); and that there exists a critical frequency above which there will always be phase lags in stationary flows with finite mean velocity. Apelt and Ledwich [5] studied numerically the transient heat transfer from a constant-temperature cylinder in a flow with a sinusoidal variation in velocity. The amplitude of the variation is 10% of the mean velocity of the flow which is at $Re_0 = 2U_0a/\nu = 10$. They found a phase lag of 0.2283π at a frequency as low

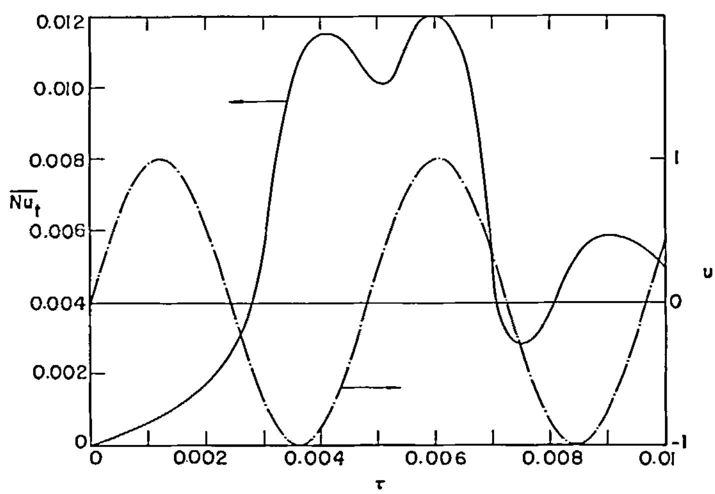

FIG. 5. Variation of fluctuating heat transfer ($\overline{Nu}_t$) with time and $\omega = 1300$, $Re_N = 10$, and $Pr = 0.73$. —·—, $\sin(\omega\tau)$.

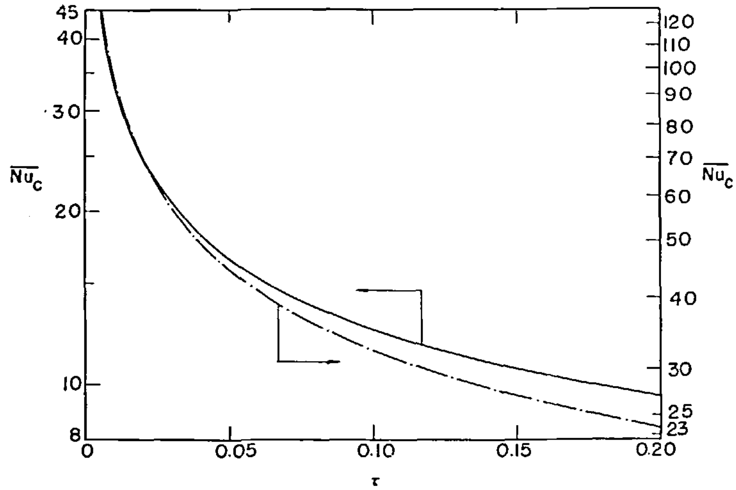

FIG. 6. Dimensionless pure-conduction heat transfer ($\overline{Nu}_c$) vs dimensionless time (τ). ——, $Pr = 0.73$; —·—, $Pr = 6.82$.

as 0.0692 Hz. Our new findings for transient flows complement the known results of Apelt and Ledwich [5] and Davis [17].

Figure 7 gives the local Nusselt number at $\theta = \pi/4$ and π for the case of an impulsively started motion. The corresponding results of Sano [2] and Jain and Goel [1] are also given in the same figure for comparison. The agreement is better near the rear stagnation point than at the forward stagnation point. The agreement seems to improve at larger values of τ. In Fig. 8, we compare the present results of Nu with those of Sano [2] at $Re_N = 100$ and $Pr = 0.73$. The agreement is only fair. However, the agreement improves when only the first two terms of equation (12) are retained in computation. Sano found that for $\tau \geqslant 0.004$ a minimum Nu exists between $\theta = 0$ and $\pi/2$. He suggested that this may be related to the flow separation. However, his numerical results show the existence of a minimum for both separated and unseparated flows. The local minimum of Nu does not seem to have been found experimentally. It should be pointed out that the boundary layer approximation used by Sano in his inner solution is probably invalid during the initial time when a boundary layer is not yet established. The present method does not require the boundary layer approximation. Figure 9 shows that the idealized model of an impulsively started uniform motion overestimates near the forward stagnation point but underpredicts near the rear stagnation point the local heat transfer from a circular cylinder which moves with a more realistic 'impulsively' started uniform velocity. Figure 10 shows the effect of Pr on Nu. Figure 11 gives some typical results for the case of a sinusoidally oscillating cylinder.

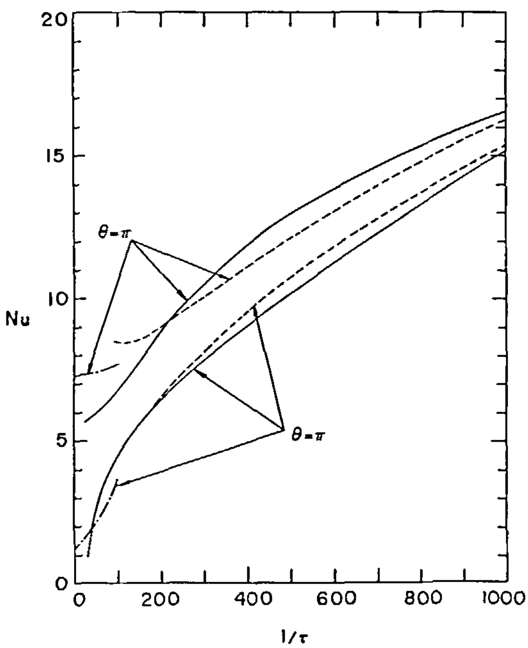

FIG. 7. Variation of local Nusselt number with time at $Re_N = 100$ and $Pr = 0.73$ for the case of impulsively started uniform motion. ——, present study; -----, Sano [2]; —·—, Jain and Goel [1].

4. CONCLUSIONS

The closed form solution we obtained can be used to determine the local as well as the net heat transfer from a constant-temperature cylinder which starts from rest any arbitrary translational fluctuation of amplitudes much smaller than the cylinder diameter. The same results may also be applied to predict the initial transient heat transfer from an impulsively started as well as more realistically possible motion of the cylinder. At given Re_N, Pr and Gr, it is shown that there exists an optimum frequency of sinusoidal oscillation for the maximum net heat transfer. Based on our numerical results on the phase lag of heat transfer and previous workers' results discussed in the last section,

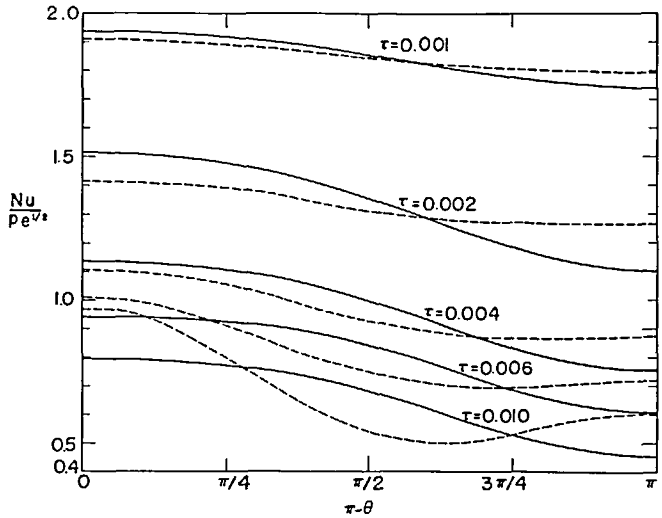

FIG. 8. Instantaneous distribution of local Nusselt number with θ at $Re_N = 100$ and $Pr = 0.73$ for the case of impulsively started uniform motion. ——, present study; -----, Sano [2].

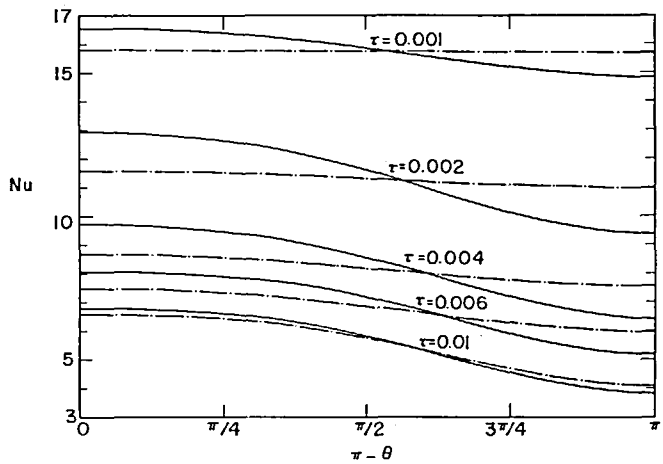

FIG. 9. Instantaneous distribution of local Nusselt number with θ at $Re_N = 100$ and $Pr = 0.73$. ——, impulsively started uniform motion; —·—, uniform acceleration until $\tau_c = 0.01$ then followed by a constant velocity.

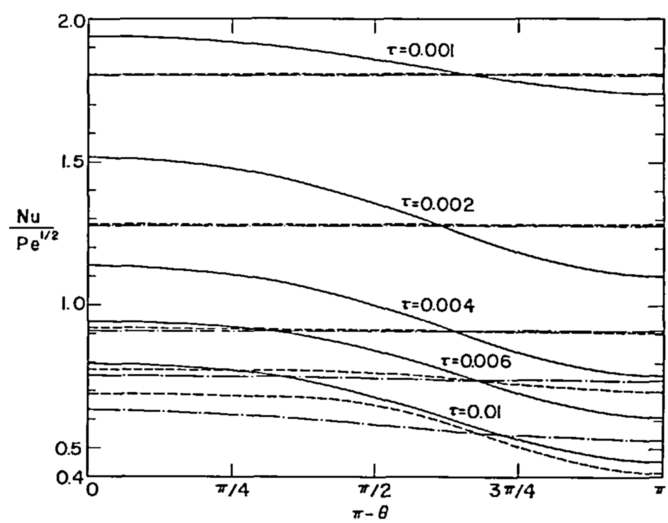

FIG. 10. Instantaneous distribution of local Nusselt number with θ at $Re_N = 100$ for the case of impulsively started uniform motion. ——, $Pr = 0.73$; -----, $Pr = 6.82$; —·—, $Pr = 10$.

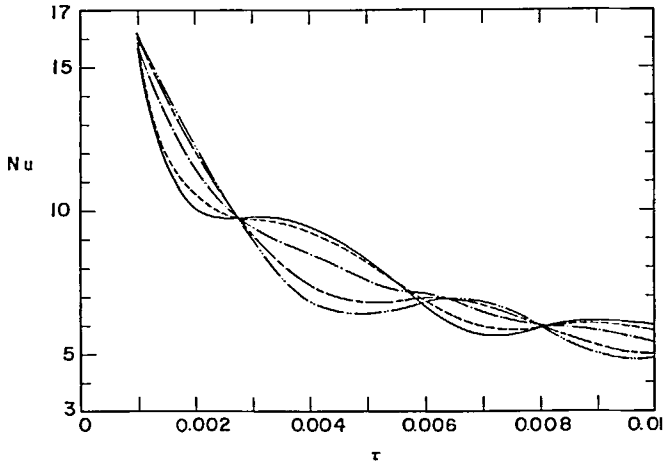

FIG. 11. Variation of local Nusselt number with time at $\omega = 1300$, $Re_N = 100$, and $Pr = 0.73$. ——, $\theta = 0$; -----, $\theta = \pi/4$; —·—, $\theta = \pi/2$; ———, $\theta = 3\pi/4$; —··—, $\theta = \pi$.

we tentatively conclude that the quasi-steady response assumed in hot-wire or hot-film calibration is invalid, since all possible frequencies of fluctuation are present in turbulent flows with or without mean velocity. This conclusion is tentative, since our results are valid only for small amplitude fluctuation while in turbulent flows the fluctuation amplitudes are mostly greater than the hot-wire diameter. Efficient numerical programs capable of predicting the net and local heat transfer in large amplitude fluctuating flows are prerequisites for making the above conclusion more definitive. Until these programs become available, the numerous experimental studies of the effects of large amplitude oscillations on heat transfer cited in the introduction will remain unsupported by theories. However, the present analytical results may serve as testing stones for the numerical accuracy of the above mentioned computer program to be developed in the near future.

Measurements of 'transient' local heat flux due to convection do not seem to exist. Finally we point out that equation (11) can be solved as an integral-differential equation to predict $u(\tau)$ with the net heat flux as measured input [22].

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APPENDIX

$$f_{ji}(r,\tau) = r^{-2}\int_1^r x_{ji}(s,\tau)s^3\,\mathrm{d}s, \quad (i,j = 1,\dot{2}),$$

$$x_{ji}(r,\tau) = \frac{1}{r}\int_1^\infty \int_0^\tau [E_1(s,r,\tau-\lambda) - V_1(s,r,\tau-\lambda)]G_{ji}(s,\lambda)s^2\,\mathrm{d}\lambda\,\mathrm{d}s,$$

$$E_1(s,r,\tau) = \frac{1}{2\tau}\exp\left[-\frac{r^2+s^2}{4\tau}\right] I_1\left[\frac{rs}{2\tau}\right],$$

$$V_1 = \int_{\gamma-i\infty}^{\gamma+i\infty} 2[K_1(vr)K_1(vs)I_1(v)/K_1(v)]\exp[v^2\tau]v\,\mathrm{d}v.$$

Where I_1 and K_1 are respectively the modified Bessel functions of the first and second kinds of the first order,

$$G_{ji}(r,\tau) = 0.5\int_1^r [F_{ji}(s,\tau)/s]\,\mathrm{d}s,$$

$$F_{11}(r,\tau) = -\frac{1}{r}\left[x_{0_r}\left(x_0 - u - \frac{u}{r^2} - \frac{1}{r^2}\int_1^r vx_0\,\mathrm{d}v\right) + x_{0_{rr}}\left(ur - \frac{u}{r} - \frac{1}{r}\int_1^r vx_0\,\mathrm{d}v\right)\right],$$

$$F_{12} = 0,$$

$$F_{2i} = -r^{-1}(f_{0_r}x_{1i_r} - f_0 x_{1i_{rr}} + x_{0_r}f_{1i_r} - x_{0_{rr}}f'_{1i}),$$

$$f'_{ji}(r,\tau) = \frac{1}{r}\int_1^r x'_{ji}(s,\tau)s\,\mathrm{d}s, \quad (i = 1,2,\ j = 1,2),$$

$$x'_{ji} = \int_1^\infty \int_0^\tau \int_0^\infty E_2(\omega,r,\tau-\lambda)C_2(\omega,r')G'_{ji}r'\omega\,\mathrm{d}\omega\,\mathrm{d}\lambda\,\mathrm{d}r',$$

$$(G'_{11}, G'_{12}) = G_1(\dot{u},\bar{g})[\Theta_0(r,\tau) - 1],$$

$$(G'_{21}, G'_{22}) = G_1\left[\left(\Theta_1, \int_1^r (\Theta_{1_\theta}/r)\,\mathrm{d}r\right)\right]\begin{pmatrix} \dot{u} & g_1 \\ -g_1 & -\dot{u}\end{pmatrix},$$

$$E_2(\omega,r,\tau) = \exp[-\omega^2\tau]C_2(\omega,r),$$

$$C_2(\omega,r) = [J_0(\omega r)Y_0(\omega) - Y_0(\omega r)J_0(\omega)]/[J_0^2(\omega) + Y_0^2(\omega)]^{1/2},$$

$$f_{2a} = r^{-1}\int_1^r x_{2a}(s,\tau)s\,\mathrm{d}s,$$

$$x_{2a} = \int_1^\infty \int_0^\tau \int_0^\infty E_2(\omega,r,\tau-\lambda)C_2(\omega,r')G_{2a}r'\omega\,\mathrm{d}\omega\,\mathrm{d}\lambda\,\mathrm{d}r',$$

$$G_{2a} = 0.5\int_1^r F_{2a}(s,\tau)\,\mathrm{d}s,$$

$$F_{2a} = r^{-1}[f_0(x_{11_r} + rx_{11_{rr}}) - f_{1_r}X_{0_r} + 2(rf_{0_r}x_{11_r} - f_1 x_{0_{rr}})],$$

$$f_{2b} = r^{-3}\int_1^r x_{2b}(s,\tau)s^5\,\mathrm{d}s,$$

$$x_{2b} = r^{-2}\int_1^\infty \int_0^\tau [E_1(r,\tau-\lambda) - V_4(s,r,\tau-\lambda)]\frac{\mathrm{d}}{\mathrm{d}\lambda}G_{2b}(s,\lambda)s^5\,\mathrm{d}\lambda\,\mathrm{d}s,$$

$$V_4 = 2\int_{\gamma-i\infty}^{\gamma+i\infty} [K_2(vr)K_2(vs)I_2(v)/K_2(v)]v\,\mathrm{d}v,$$

$$G_{2b} = 0.5\int_1^r s^{-1}F_{2b}(s,\tau)\,\mathrm{d}s,$$

$$F_{2b} = F_{2a} - 4(rf_{0_r}x_{11_r} - f_{11}x_{0_{rr}}),$$

$$f_{23} = \int_1^r s^{-1}x_{23}(s,\tau)\,\mathrm{d}s,$$

$$x_{23} = r\int_1^\infty \int_0^\tau [E_1(r,\tau-\lambda) - V_1(s,r,\tau-\lambda)]\frac{d}{d\lambda}[G_{23}(s,\lambda)]s^2\,d\lambda\,ds,$$

$$G_{23} = 0.5\int_1^r F_{23}(s,\tau)s\,ds,$$

$$F_{23} = r^{-1}(f_{0_r}x_{12_r} + f_0 x_{12_{rr}} - f'_{12_r}x_{0_r} - f'_{12}x_{0_{rr}}),$$

$$J(r,\tau) = \int_1^\infty \int_0^\alpha [E_1(r,\alpha-\lambda) - V_1(r,s,\alpha-\lambda)]H(s,\lambda)s\,d\lambda\,ds,$$

$$H = (2/\pi r)f_0(r,\tau)\int_0^\infty \exp(-\omega^2\tau/Pr)C_1(\omega,r)\,d\omega,$$

$$C_1(\omega,r) = [Y_1(\omega r)J_0(\omega) - J_1(\omega r)Y_0(\omega)]/[J_0^2(\omega) + Y_0^2(\omega)].$$

$$X_i(r,\tau) = \int_1^\infty \int_0^\alpha [E_1(r,\alpha-\lambda) - V_1(s,r,\alpha-\lambda)]G_i(s,\lambda)\,d\lambda\,ds,$$

$$G_i(r,\tau) = f'_{1i}\Theta_{0_r}, \quad (i = 1,2),$$

$$(M,N) = \int_1^\infty \int_0^\alpha \int_0^\infty E_2(\omega,\alpha-\lambda)[C_M(r',\lambda), C_N(r',\lambda)]C_2(\omega,r')\omega\,d\omega\,d\lambda\,dr',$$

$$C_M = f_0 J_r, \quad C_N = J f_{0_r},$$

$$Q = \int_1^\infty \int_0^\alpha \left\{\frac{1}{4(\alpha-\lambda)^2}\exp\left[-\frac{r^2+s^2}{4(\alpha-\lambda)}\right]\left(sI_0\left[\frac{rs}{2(\alpha-\lambda)}\right] - \left[r + \frac{4(\alpha-\lambda)}{r}\right]\right.\right.$$
$$\left.\left.\times I_1\left[\frac{rs}{2(\alpha-\lambda)}\right]\right) + \left[\frac{1}{r}V_1(s,r,\alpha-\lambda) - V_{1_r}(s,r,\alpha-\lambda)\right]\right\}B(s,\lambda)s^2\,d\lambda\,ds,$$

$$B(r,\tau) = \int_1^r s^{-2}\{C_Q(s,\tau) - 2r^{-1}[M(s,\tau) - N(s,\tau)]\}\,ds,$$

$$C_Q(r,\tau) = 2f_{11}(r,\tau)\tau_{0_r}(r,\tau)/Pr.$$

TRANSFERT THERMIQUE VARIABLE A PARTIR D'UN FIL DANS UN ENVIRONNEMENT FLUCTUANT VIOLEMMENT

Résumé—Une solution analytique de perturbation est obtenue pour le problème du transfert thermique transitoire à partir d'un cylindre circulaire à température constante dans un écoulement de fluide newtonien incompressible fluctuant violemment. Le paramètre utilisé est le rapport de l'amplitude maximale de la fluctuation au diamètre du cylindre. La solution est valable asymptotiquement pour une valeur quelconque des nombres de Reynolds et de Prandtl. La convection naturelle est un effet du second ordre. La solution est aussi appliquée au transfert thermique initial d'un fil qui part du repos pour un mouvement de translation quelconque. Des résultats numériques en fonction des nombres de Nusselt locaux, variables par rapport au temps, et moyens sont utilisés pour montrer le déphasage de la réponse thermique à la fluctuation de vitesse. On montre qu'il existe une fréquence optimale de l'oscillation sinusoïdale, pour un transfert thermique maximal avec des paramètres donnés de l'écoulement.

INSTATIONÄRER WÄRMEÜBERGANG AN EINEM DRAHT IN EINER STARK FLUKTUIERENDEN UMGEBUNG

Zusammenfassung—Für das Problem des instationären Wärmeübergangs an einem kreisförmigen Zylinder konstanter Temperatur in einer stark fluktuierenden Strömung von nichtkompressiblen Newtonschen Flüssigkeiten, wu.de mittels Störungsansatz eine geschlossene Lösung erhalten. Parameter in der Lösung ist das Verhältnis von maximaler Amplitude der Fluktuationen zum Durchmesser des Zylinders. Die Lösung ist für alle Werte der Prandtl- und Reynoldszahlen asymptoisch gültig. Die freie Konvektion wurde als Einfluß zweiten Grades berücksichtigt. Mit der Lösung kann auch der anfänglich instationäre Wärmeübergang an einem Draht, der aus der Ruhe heraus eine beliebige translatorische Bewegung beginnt, berechnet werden. Numerische Ergebnisse in Form von zeitabhängigen örtlichen und mittleren Nusseltzahlen werden verwendet, um die kennzeichnende Phasenverschiebung des Wärmeübergangs gegenüber der fluktuierenden Geschwindigkeit darzustellen. Es wird gezeigt, daß eine optimale Frequenz bei sinusförmiger Schwingung für maximalen Wärmeübergang bei vorgegebenen Strömungsparametern existiert.

НЕУСТАНОВИВШИЙСЯ ТЕПЛОПЕРЕНОС ОТ ПРОВОЛОКИ В СИЛЬНО ПУЛЬСИРУЮЩЕЙ ОКРУЖАЮЩЕЙ СРЕДЕ

Аннотация—В замкнутом виде получено решение в возмущениях для задачи неустановившегося переноса тепла от кругового цилиндра с постоянной температурой в сильно пульсирующих потоках несжимаемых ньютоновских жидкостей. Используемый в решении малый параметр представляет собой отношение максимальной амплитуды колебаний к диаметру цилиндра. Решение является асимптотически справедливым при любых значениях чисел Прандтля и Рейнольдса. Естественная конвекция рассматривается как эффект второго порядка. Решение может использоваться также для расчета неустановившегося переноса тепла от цилиндра, совершающего произвольное поступательное движение из состояния покоя. Численные результаты в виде зависящих от времени локальных и средних чисел Нуссельта используются для демонстрации влияния пульсаций скорости на тепловую инерцию цилиндра. Показано, что для максимального результирующего теплового потока при заданных параметрах течения характерна определенная оптимальная частота синусоидальных колебаний цилиндра.